Recent Advances in Random Vibrations of Nonlinear Systems for Reliability, Durability and Accelerated Testing

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August 30, 2018

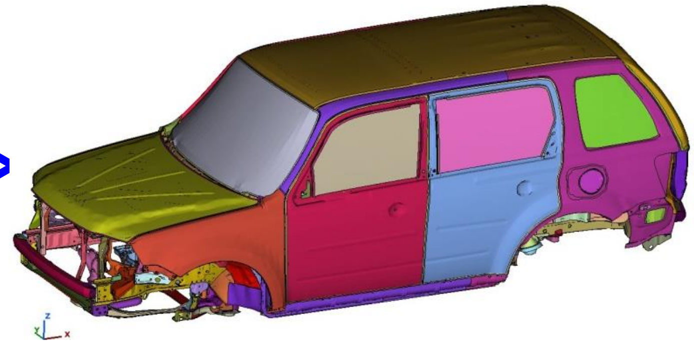
Overview

- Motivation: Reliability, Durability, ALT
- Background Theory
  - Characterization of input and output process (linear and nonlinear vibratory systems)
  - Fatigue Reliability/Durability
- Fatigue Life Estimation of Nonlinear Vibratory Systems under Gaussian or Non-Gaussian Loading
- Accelerated Life Testing (ALT)
  - Enhanced ALT design using Saddlepoint Approximation (SPA)
- Summary and Conclusions

Motivation (Reliability, Durability, ALT)

Excitation ⇨

Commonly Non-Gaussian



Nonlinear, Large Size (>15M DOFs)

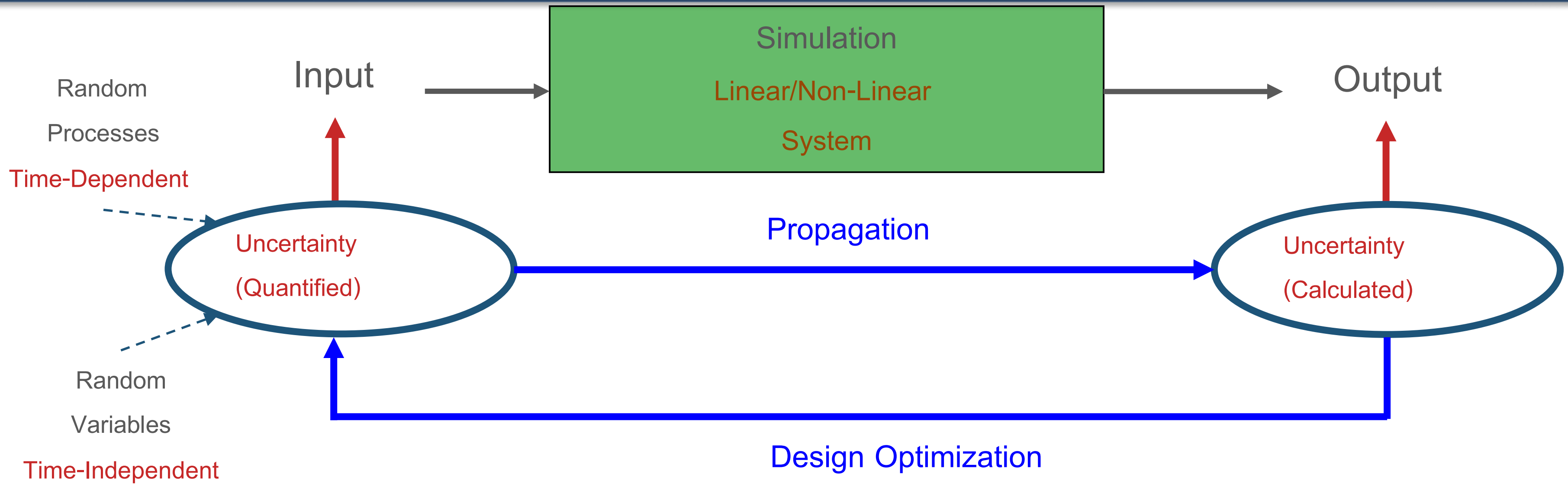
⇨ **Vibratory Response**

Vibration Analysis is Computationally Intensive

Deterministic Design Optimization Requires <u>MANY</u> Vibration Analyses

Probabilistic Design Optimization Requires <u>EVEN MORE</u> Vibration Analyses

Design Under Uncertainty



Challenges:

- Quantification of input random processes (Gaussian / Non-Gaussian)
- Propagation of Uncertainty to characterize output random processes

  (Linear/Nonlinear System – Gaussian / Non-Gaussian processes)

  - How to reduce the number of system simulations?
  - How to reduce computational cost of each simulation for very large vibratory systems (millions of DOFs) ?

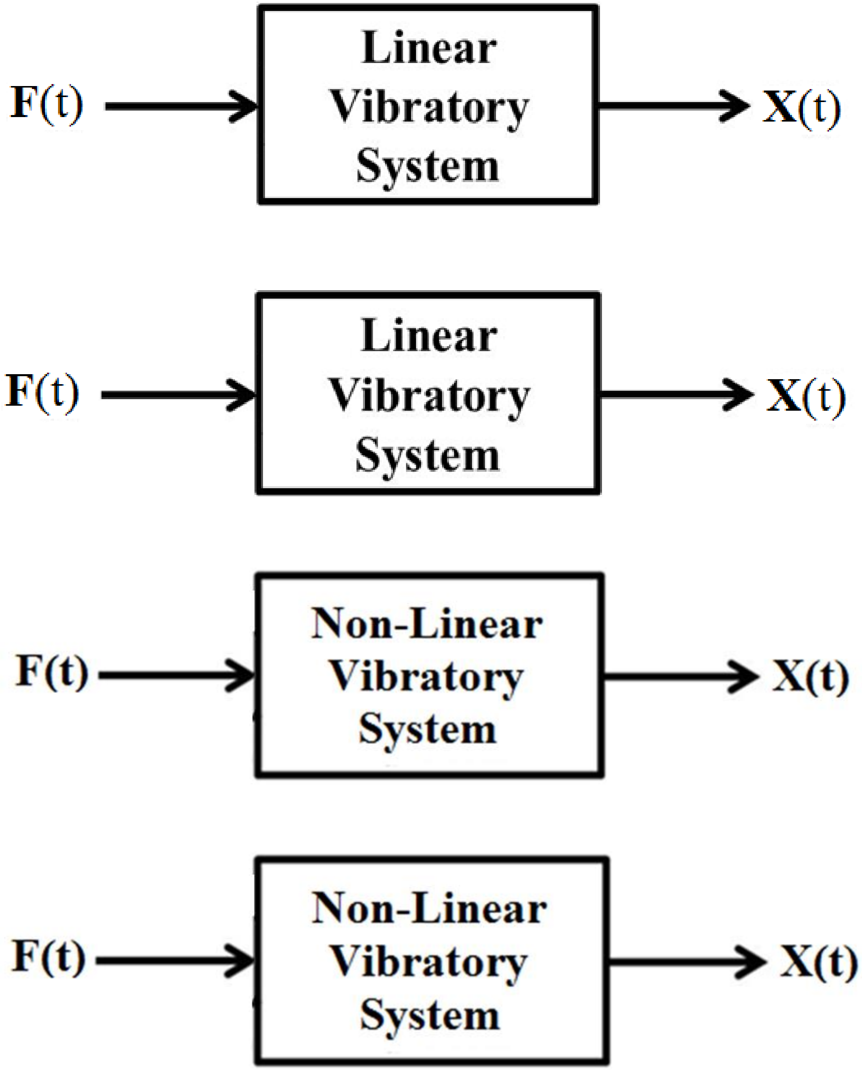
Input-Output Relationships for Vibratory Systems

Case 1: Gaussian → Linear Vibratory System → Gaussian

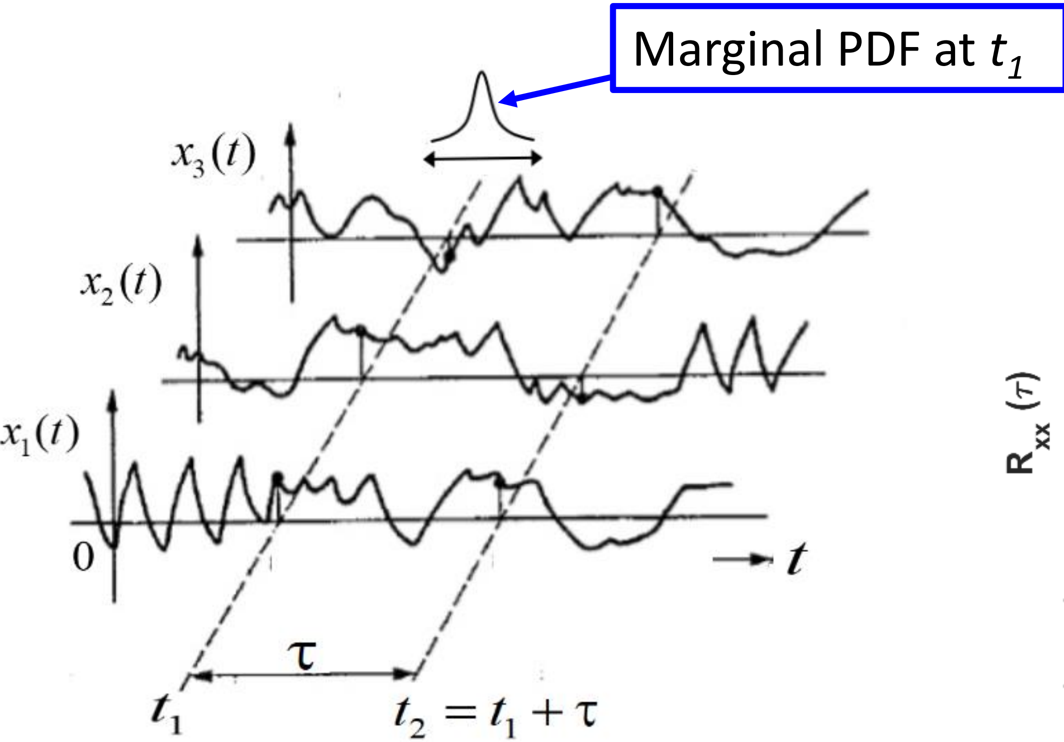
Case 2: Non-Gaussian → Linear Vibratory System → Non-Gaussian

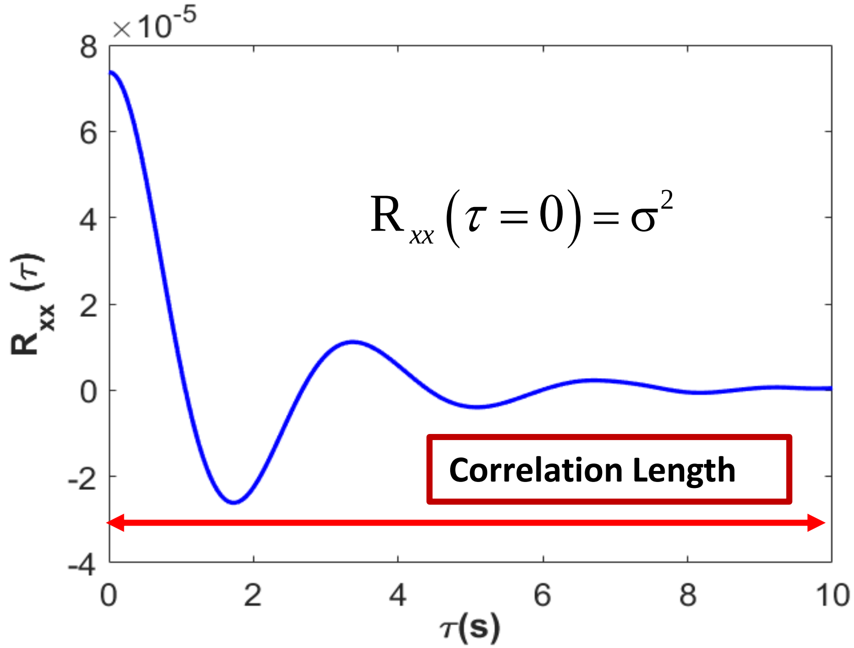
Case 3: Gaussian → Non-Linear Vibratory System → Non-Gaussian

Case 4: Non-Gaussian → Non-Linear Vibratory System → Non-Gaussian



Characterization of a Random Process

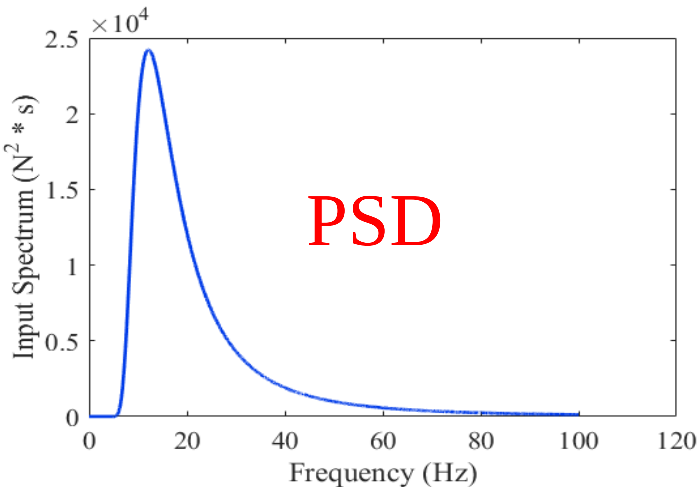




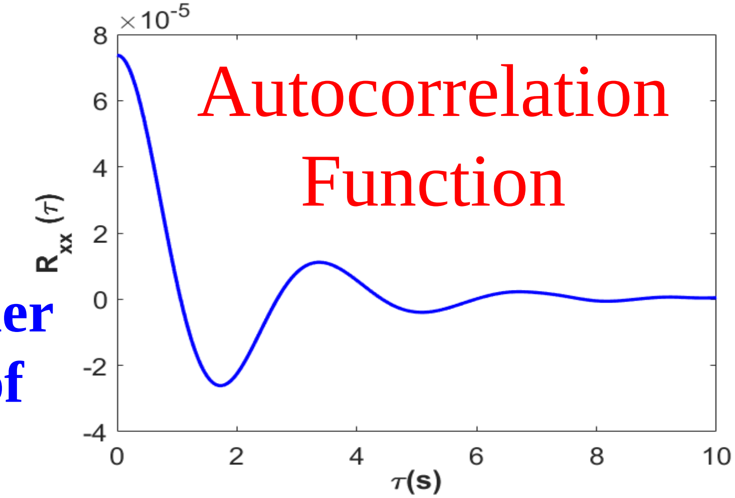
A zero mean, stationary Gaussian process is fully characterized by its autocorrelation function.

For a non-Gaussian process we need skewness and kurtosis in addition to the autocorrelation function.

Simulation of a Gaussian Process in Time Domain



Inverse Fourier Transform of PSD



- Discretize the autocorrelation function and form the correlation matrix
- Obtain eigenvalues and eigenvectors of correlation matrix and use them in a Karhunen-Loeve (K-L) expansion

K-L Expansion

$$\xi(t) = \sum_{i=1}^{N} \sqrt{\lambda_i} \cdot f_i(t) \cdot \xi_i$$

λ_i: Eigenvalues of correlation matrix

$f_i(t)$: Eigenvectors of correlation matrix

ξ_i: Independent standard normal random variables

The number of required terms *N* increases fast with increasing simulation time

Simulation of a Non-Gaussian Process (PCE-KL)

Express Non-Gaussian Process Z(t) in terms of Gaussian Process ξ(t) (PCE)

$$Z(t)=\sum_{i=0} b_i(t)\Psi_i(t)=b_0(t)+b_1(t)\xi(t)+b_2(t)\left(\xi^2(t)-1\right)$$
$$+b_3(t)\left(\xi^3(t)-3\xi(t)\right)+b_4(t)\left(\xi^4(t)-6\xi^2(t)+3\right)+\Lambda$$

b_i: coefficients to be calculated

ξ(t): Standard Normal Process

Use orthogonality of Hermite polynomials to calculate $C_{\xi\xi}(t_1,t_2)$

$$C_{ZZ}(t_1,t_2)=\sum_{i=1} b_i(t_1)b_i(t_2)\cdot(i!)\cdot\left(E\left[\xi(t_1)\xi(t_2)\right]\right)^i$$

$C_{ZZ}(t_1,t_2)$: Covariance

Known (given)

Unknown (Calculate) $C_{\xi\xi}(t_1,t_2)=E\left[\xi(t_1)\xi(t_2)\right]$

K-L Expansion

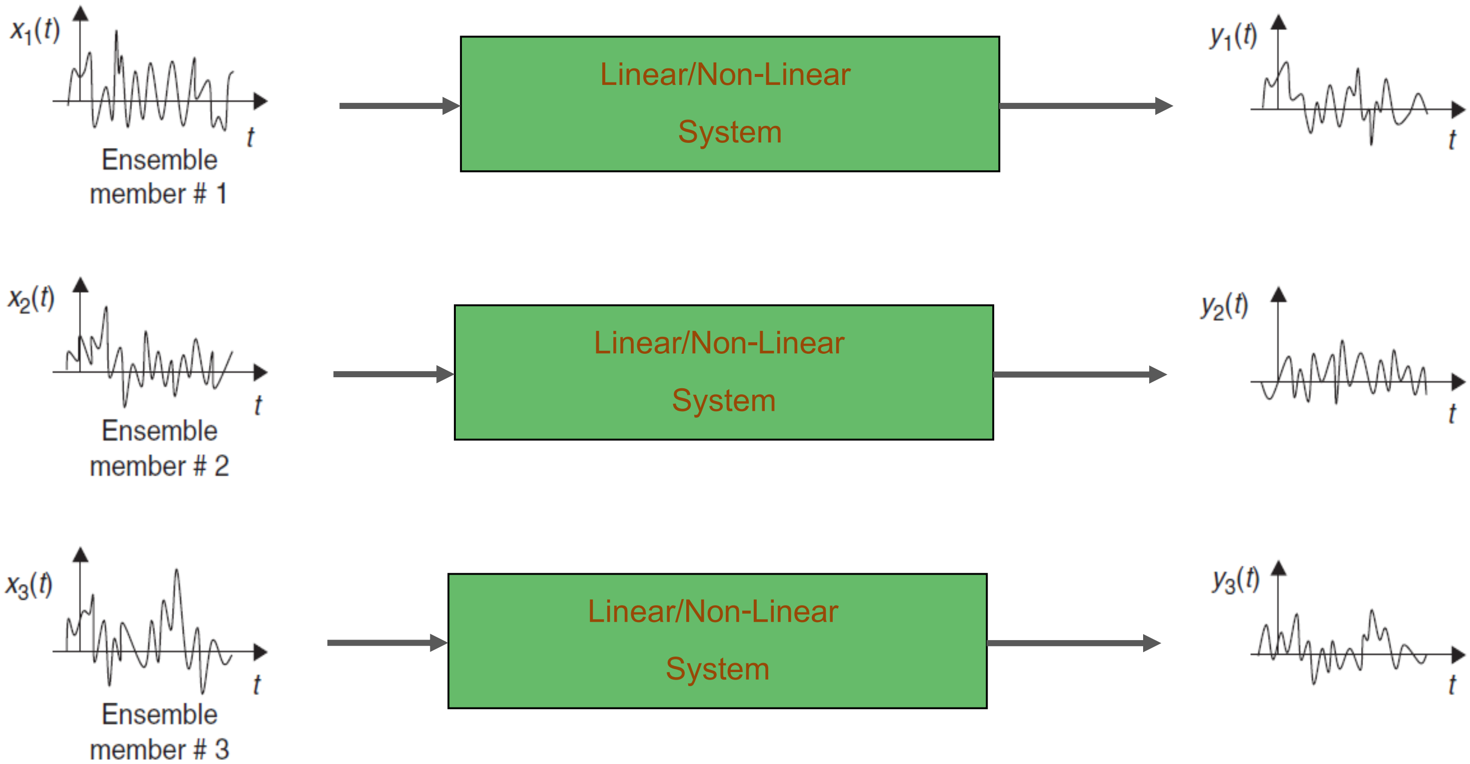
$$\xi(t)=\sum_{i=1}^{N}\sqrt{\lambda_i}\cdot f_i(t)\cdot\xi_i$$

λ_i: Eigenvalues of $C_{\xi\xi}(t_1,t_2)$

f_i(t): Eigenvectors of $C_{\xi\xi}(t_1,t_2)$

ξ_i: Independent standard normal variables

Random Vibrations: Fundamental Tasks



- Sample functions (trajectories) must be generated for the input process (PCE – KL)
- Trajectories of output process must be calculated and used to quantify the output process (QMC). For efficiency, the simulation (calculation of output) time must be short (e.g. as long as the process correlation length)

Characterization of the Output (Gaussian) Process for a Linear System

Case 1:

Gaussian

Gaussian

➢ A linear vibratory system is fully characterized by its **FRF** (Frequency Response Function)

$$S_{XX}(\omega) = |H(\omega)|^2 S_{FF}(\omega)$$

PSD of Output — FRF — PSD of Input

➢ **Linear Vibratory System**

➢ **<u>Stationary & Gaussian</u> Input**

- An Inverse Fourier Transform of $S_{XX}(\omega)$ yields the output autocorrelation function
- KL expansion using the output autocorrelation function yields trajectories of the output process

Characterization of the Output (Non-Gaussian) Process for a Nonlinear System

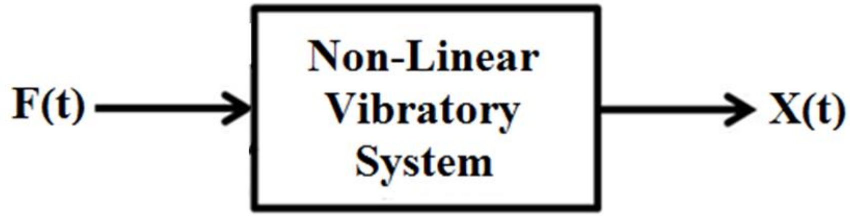
Case 2:

Non-Gaussian

Non-Gaussian

Case 3:

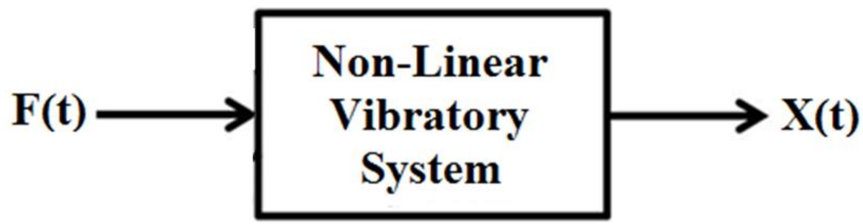
Gaussian



Non-Gaussian

Case 4:

Non-Gaussian



Non-Gaussian

Characterization of the Output (Non-Gaussian) Process for a Nonlinear System

- How can we use the calculated trajectories of the output process to characterize it accurately and efficiently?
- How many trajectories do we need? The number of trajectories must be small for efficiency.

A Quasi Monte Carlo (QMC) approach is used

1. The N-dimensional $(\xi_1, \xi_2, \ldots \xi_N)$ space is space-filled with M points
2. For each of the M points, a K-L trajectory $\xi(t) = \sum_{i=1}^{N} \sqrt{\lambda_i} \cdot f_i(t) \cdot \xi_i$ of the input process is generated and the corresponding output trajectory is obtained
3. The above M output trajectories are used to estimate the 4 moments and autocorrelation function of the output process (QMC method)
4. Finally PCE-KL is used to characterize the output process

Features of Methodology (Fatigue / Durability)

Linear or Non-Linear
system random parameters

Miner's Damage Rule
SN-curve random parameters

Step 1: F(t)

Step 2:
Vibratory
System

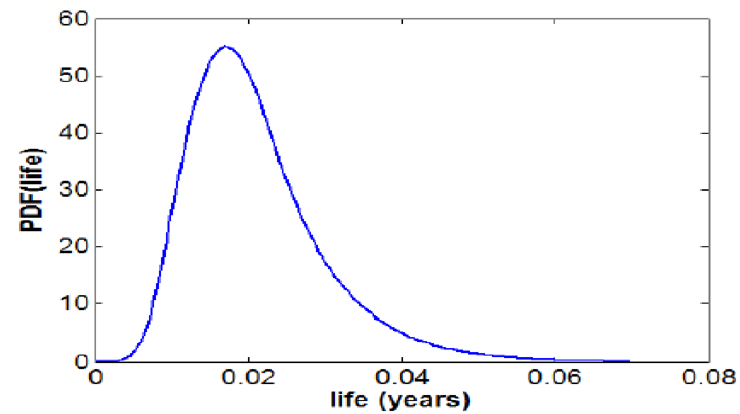
Step 3: S(t)

Step 4:
Fatigue
Model

Step 5: Life

<u>Input Excitation</u>
Stationary/
Non-stationary
Gaussian/
Non-Gaussian

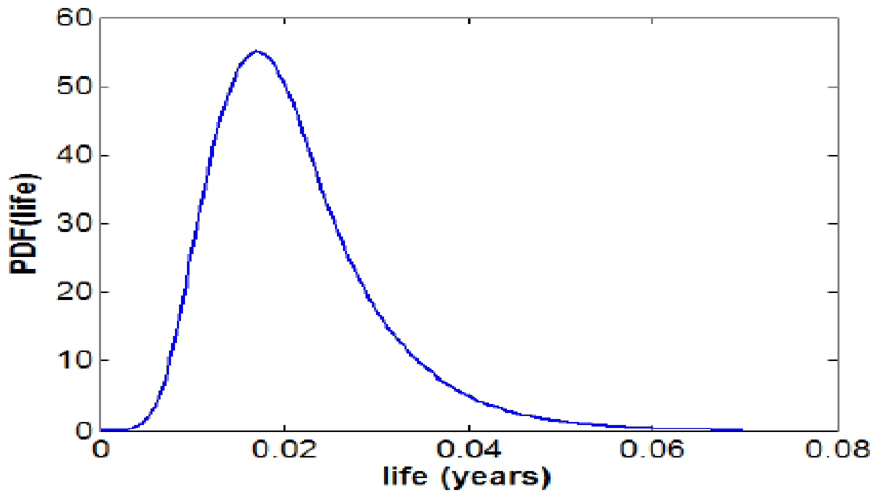
PDF of life



Features of Methodology (Fatigue / Durability)

Fatigue reliability:

- Characterize the output stress process **(PCE-KL-QMC)**
- Generate new output trajectories without solving the system
- Rainflow counting method to extract cycles
- Miner's rule to calculate the cumulative damage
- SPA to calculate the **PDF of life**



Features of Methodology (Fatigue / Durability)

Design for fatigue:

The system uncertainty comes from:

- **System parameters** (e.g. stiffness of vehicle suspension)
- **Material properties** (e.g. S-N curve parameters)

Our **goal**:

- Modify mean values of system parameters to achieve a fatigue life target with high probability

⇒ **Reliability-Based Design Optimization (RBDO)**

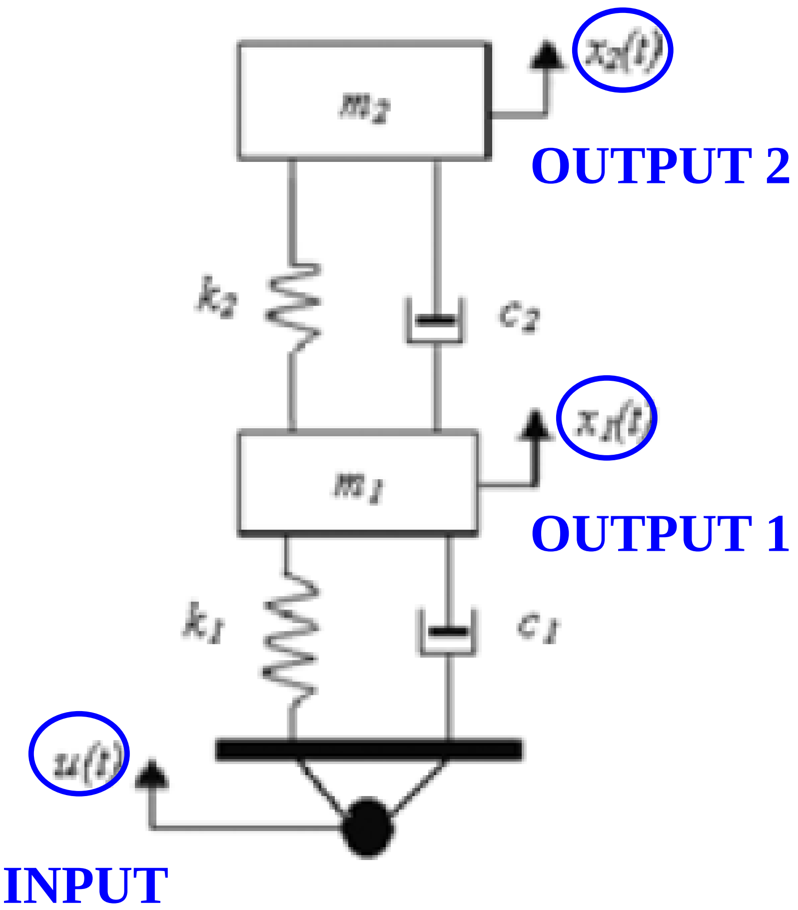
Features of Methodology (ALT)

Accelerated Life Testing (ALT):

- Characterize Gaussian or non-Gaussian input excitation (e.g. **PCE-KL**)
- Reduce uncertainty of fatigue life prediction by performing tests at higher stress levels
- An optimization problem minimizes the cost of ALT testing while improving the accuracy of fatigue life prediction

Assumptions on Life Distributions and Stress-Life relationships are **lifted**

Quarter Car Example: Fatigue Reliability



Equations of Motion

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2 x_2 = k_1 u + c_1\dot{u}$$

$$m_2\ddot{x}_2 - c_2(\dot{x}_1 - \dot{x}_2) - k_2(x_1 - x_2) = 0$$

Input Process u(t)

Non-Gaussian road elevation

Output Stress Process τ(t)

Function of spring deflection

$(x_1 - x_2)$

Characterization of Input Process u(t)

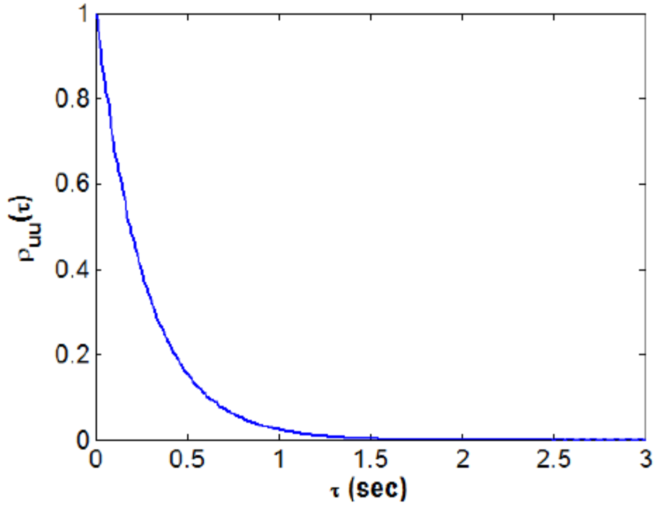
First 4 statistical moments

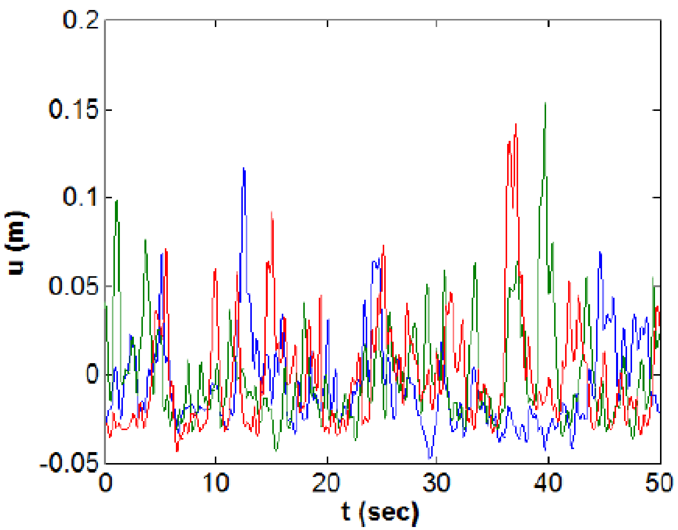
Quantity	Description	Value
μ_1	Mean	0
$m_2 (= \sigma^2)$	Variance	0.001
m_3 / σ^3	Skewness Coefficient	2
m_4 / σ^4	Kurtosis Coefficient	10

Note: Same μ_1, m_2, PSD with a Gaussian road elevation according to **ISO 8608** standard

Create PCE-KL stochastic metamodel for input process and generate trajectories of u(t)

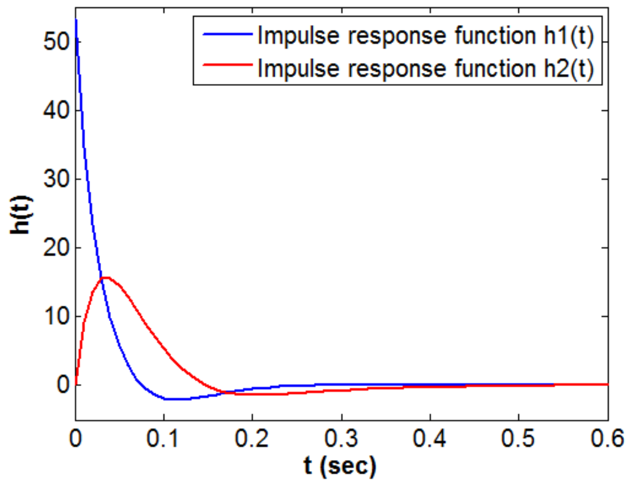
Autocorrelation



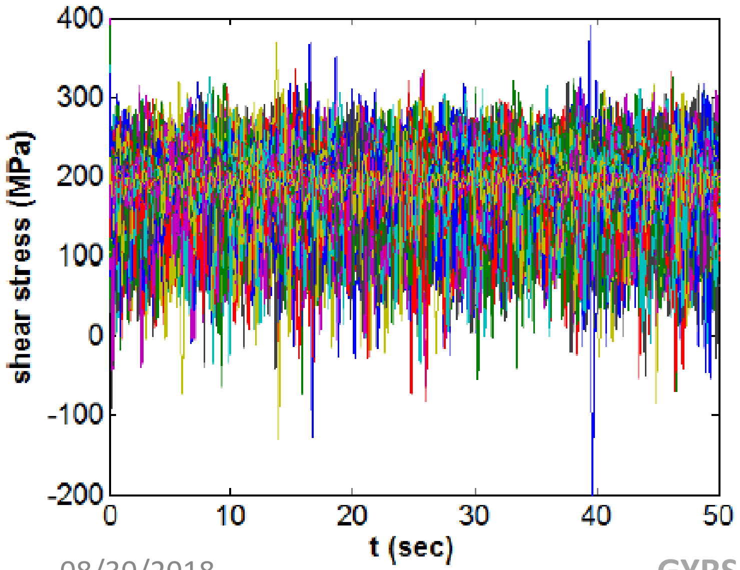


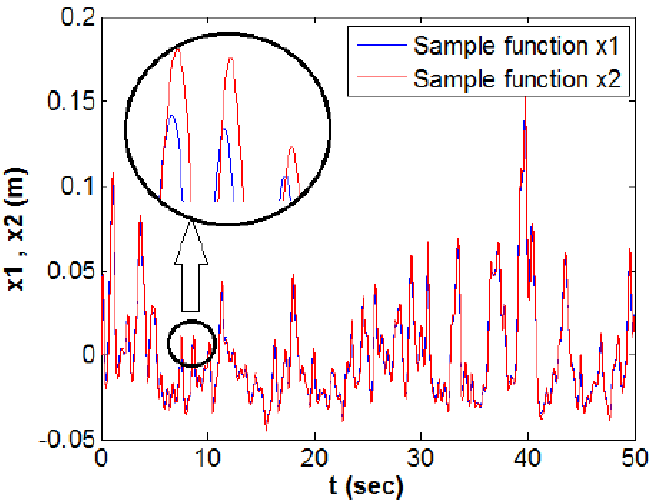
Characterization of Output Stress Process τ(t)

- Calculate the 4 moments & autocorrelation of output displacement processes $x_1(t)$, $x_2(t)$



- Develop PCE-KL stochastic metamodel for output stress process τ(t)
- Generate new trajectories of τ(t) using the metamodel

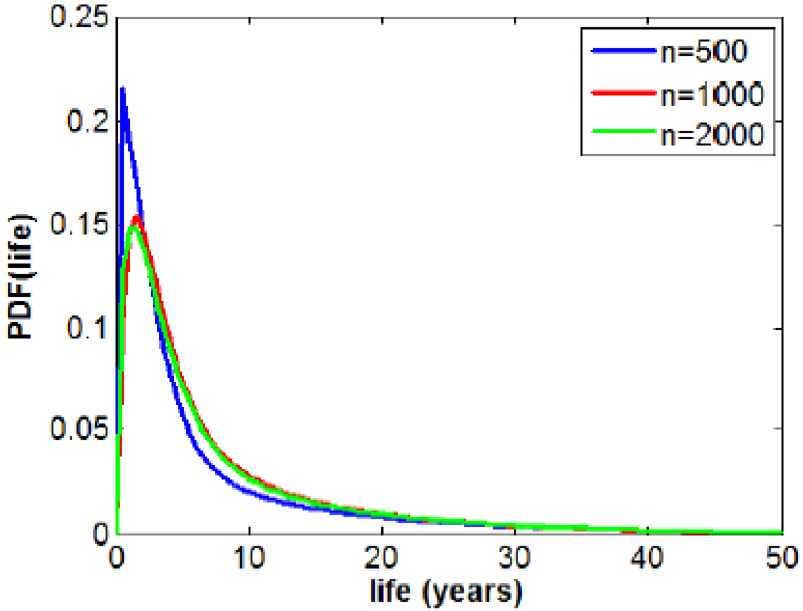




Estimation of PDF of Fatigue Life

- Each output stress trajectory is rainflow counted and then Miner's rule is used to get a realization of cumulative damage **D**

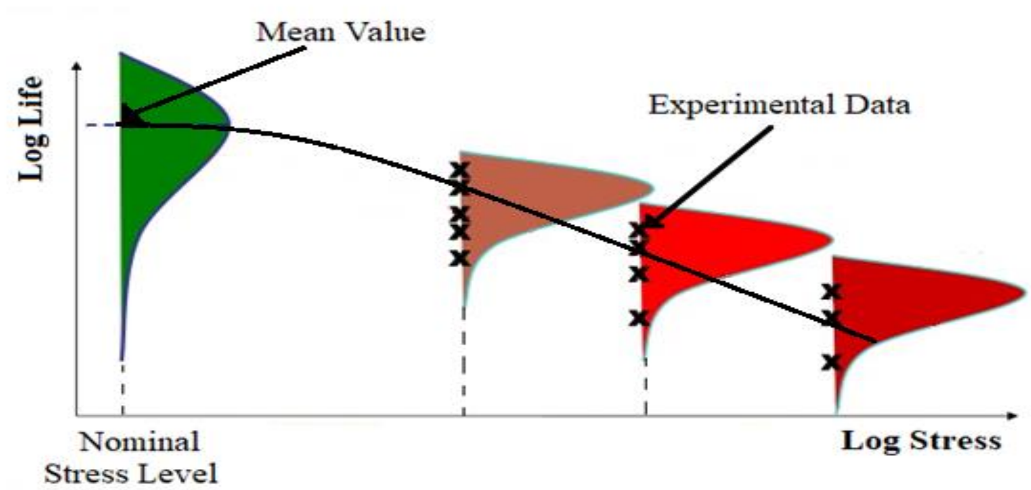
- SPA calculates the PDF of fatigue life (=1/D) using the available realizations D_i

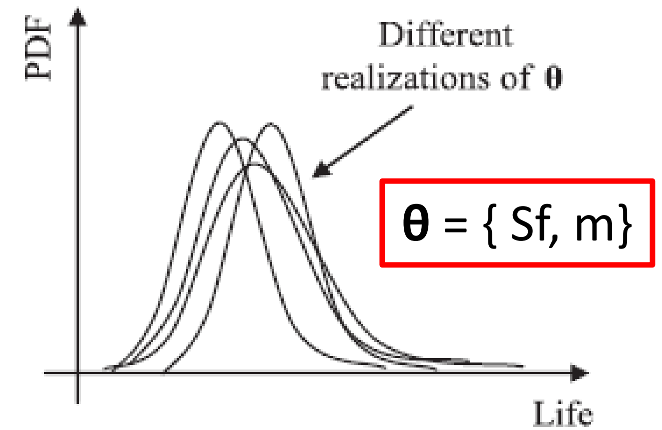


Convergence is achieved for 1000 trajectories of output stress process

Note: This approach predicts the **entire PDF of fatigue damage** (not only the E(D))

Current Accelerated Life Testing (ALT) Process





Uncertainty in S-N curve parameters $\mathbf{S_f}$ and **m**

Steps of current ALT design process

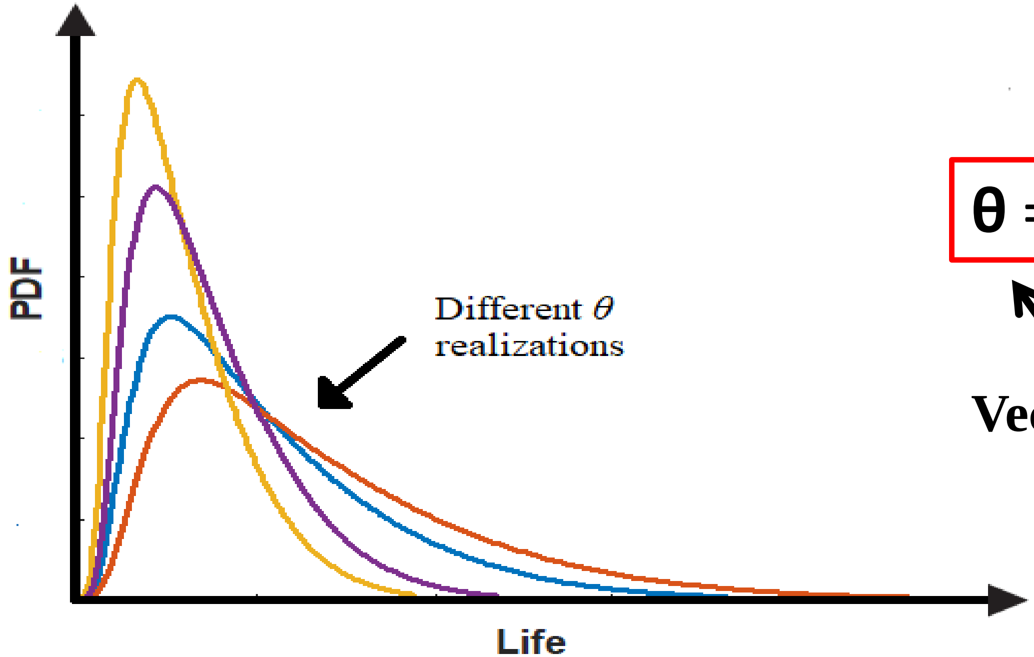
Parameters determined from experiments

1. Determine the stress levels
2. Assume a life distribution (Weibull, Lognormal, etc.)
3. Assume a stress-life relationship $\mu_T(S) = \alpha_1 + \alpha_2 \log(S)$
4. Estimate distribution parameters (MLE) using test data
5. Calculate product reliability R_{S_0} at nominal conditions

Enhanced ALT Design using SPA

1) Calculate the PDF of fatigue life at elevated stress level using SPA for different realizations of uncertain parameters (i.e. S_f and m of S-N curve)

No assumption of **underlying life distribution** is made

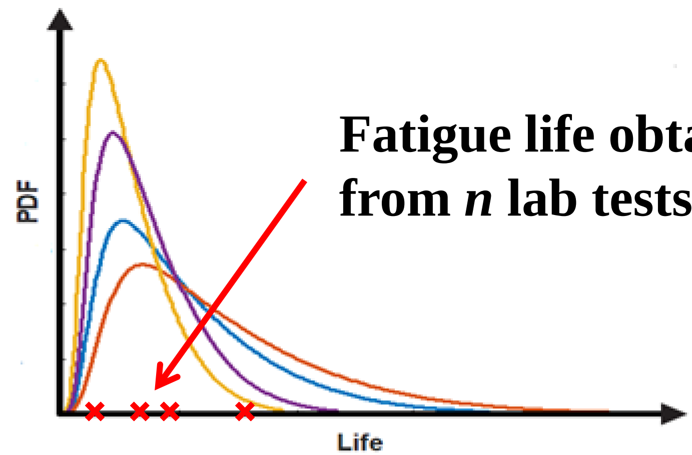


$\boldsymbol{\theta} = \{ S_f, m\}$

Vector of random variables

Enhanced ALT Design using SPA

2) Formulate a Likelihood Function to estimate the uncertain model & material parameters



$$L(\boldsymbol{\theta}) = f(x_1;\boldsymbol{\theta})\cdot f(x_2;\boldsymbol{\theta})\cdot f(x_3;\boldsymbol{\theta})\ldots f(x_n;\boldsymbol{\theta})$$

θ: Vector of uncertain model & material parameters

$f_i(.)$: PDF of fatigue life using SPA (Saddlepoint Approximation)

x_i: Life for ith experiment, $i=1,2,\ldots,n$

Enhanced ALT Design using SPA

3) Solve following optimization problem to obtain the MLE values of the uncertain model & material parameters

$$\max_{\boldsymbol{\theta},\mathbf{n}} L(\boldsymbol{\theta})$$ → Calculated using **SPA**

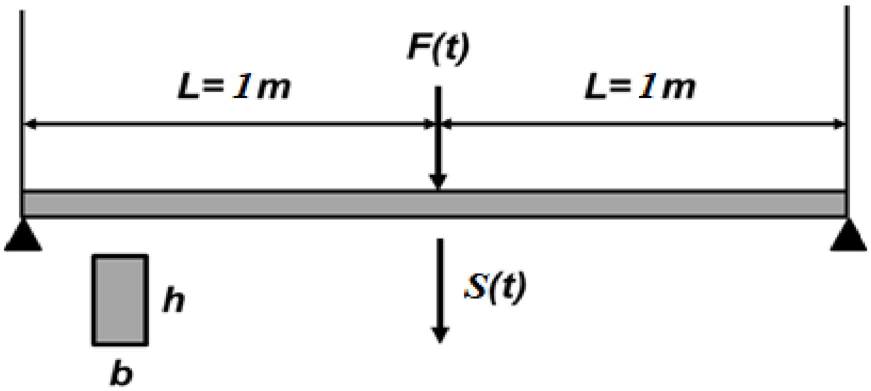
$$s.t \quad 1 \leq \mathbf{n} \leq n_{max}$$ → Number of lab experiments. The n_{max} depends on the **available budget** for lab tests

$$\boldsymbol{\theta}_{\min} \leq \boldsymbol{\theta} \leq \boldsymbol{\theta}_{\max}$$ → Vector of uncertain model & material parameters

4) Use Fisher information theory to obtain the confidence interval of θ

5) Calculate reliability of the system at nominal conditions

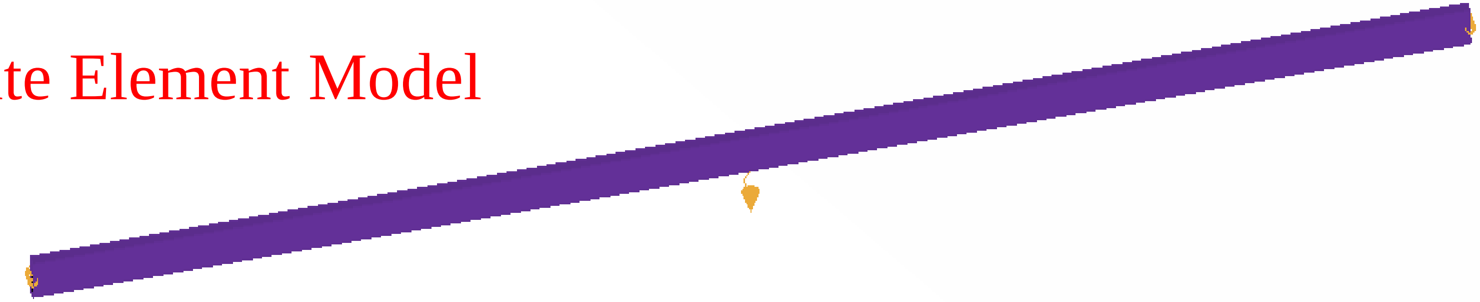
ALT Example: Simply Supported Beam



F(t):
Input Load process (Gaussian)

S(t):
Output Stress process

Finite Element Model



Uncertain Model & Material Parameters

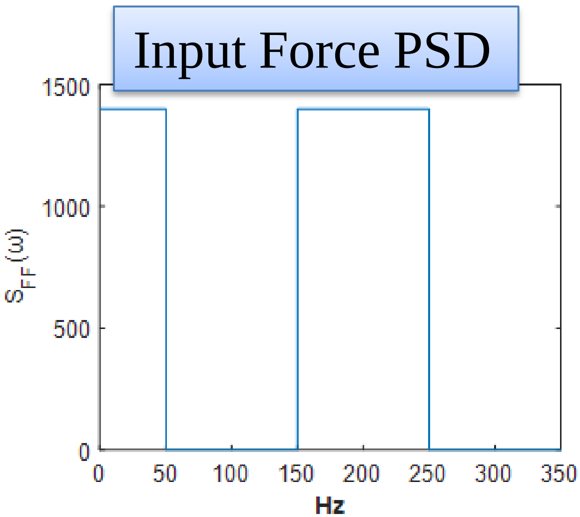
$\boldsymbol{\theta} = \{S_f, m\}$

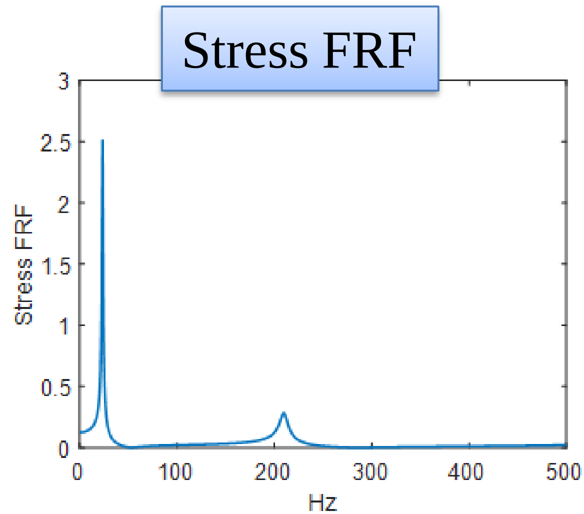
(parameters of S-N curve)

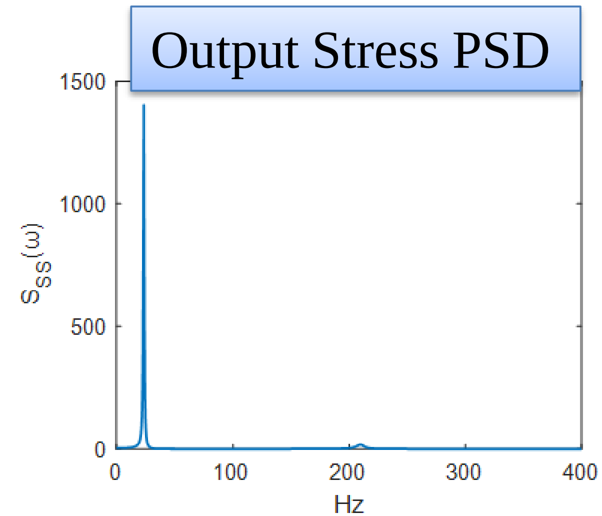
Variable	Mean	Standard Deviation	Distribution
S_f	1298	129.8	Gaussian
m	5.56	0.556	Gaussian

ALT Example: Simply Supported Beam

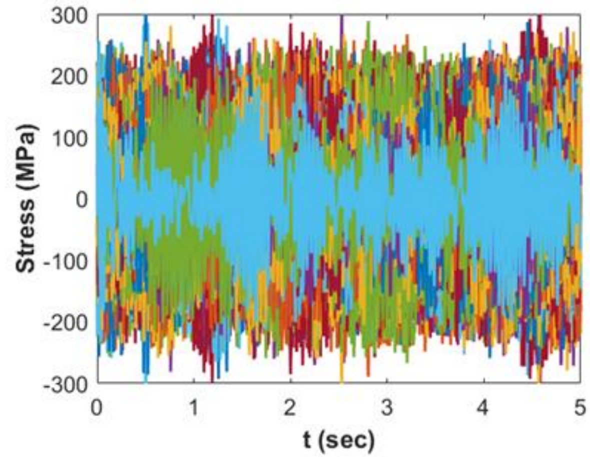
Linear System







Inflated, without violating the failure modes of the system

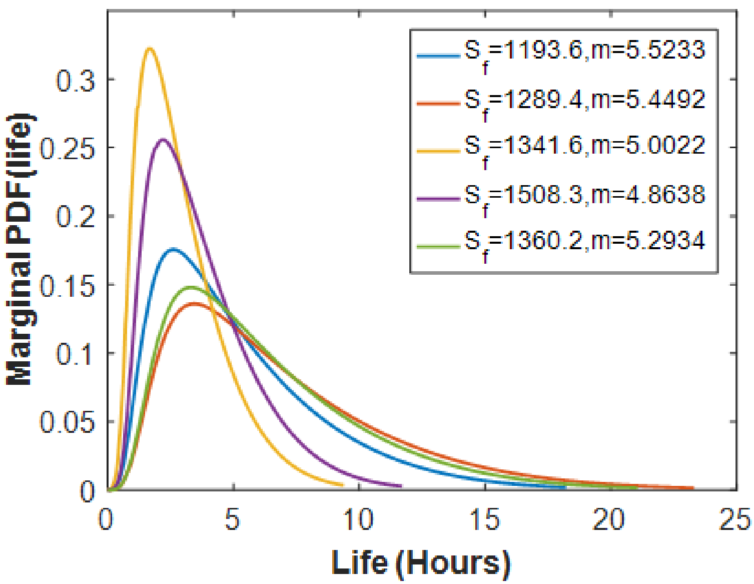


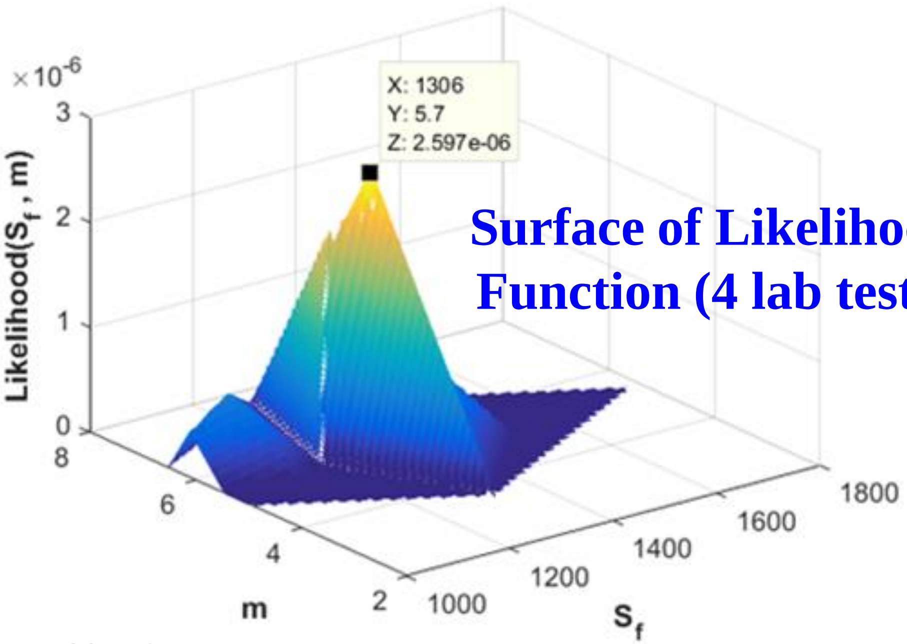
Generate stress trajectories

(Karhunen-Loeve expansion)

ALT Example: Simply Supported Beam

- For a random sample of (S_f , m), <u>each</u> stress trajectory is rainflow counted and Miner's rule is applied to estimate fatigue life.
- Saddlepoint approximation estimates the PDF of fatigue life for the (S_f , m) sample. The respective likelihood value is calculated for a given number of tests.
- Optimization determines the optimum value of (S_f , m) and number of tests *n*.





Surface of Likelihood Function (4 lab tests)

ALT Design Optimization Results

➢ Fisher Information theory provides the confidence interval of estimated model and material properties

# Experiments	$\widehat{S_f}$	$\hat{m}$
1	1400.42	5.4344
2	1400.36	5.3481
3	1399.61	5.4340
...	...	...
9	1306.56	5.6348
10	1306.56	5.6700
11	1306.56	5.6700
12	1306.56	5.6700
20	1306.56	5.6700
50	1306.56	5.6700

Confidence Intervals

# Experiments	S_f		m	
	90% Confidence Interval			
	Lower	Upper	Lower	Upper
1	0	2846.1	2.69	8.65
2	688.4	1923.6	4.45	6.89
3	528.0	2084.0	4.17	7.17
...	...	...	...	...
9	555.7	2056.3	4.24	7.10
10	673.4	1938.6	4.46	6.88
20	1126.8	1485.2	5.32	6.02
30	1149.9	1462.1	5.37	5.97
40	1180.9	1431.1	5.42	5.92
50	1268.1	1343.9	5.61	5.73

10 experiments are enough to obtain the calibrated MLE values of uncertain parameters

Summary & Conclusions

- A novel random vibrations approach for **vibratory systems** excited by **non-Gaussian** processes was presented using PCE, KL and QMC.
  - Applications in reliability and durability

- An **ALT methodology** to estimate fatigue life was presented.
  - **No assumption** for the **life distribution** and **stress-life relationship**
  - The **uncertainty** in **model** and **material properties** is **considered**

- Examples demonstrated the presented approaches

Thank you for your attention

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Q & A